Astronomy & Astrophysics manuscript no. proj[11]
December 13, 2007

Mass determination of the two components of BKPeg

Spectroscopic analysis of a binary star system

D. Juncher, K. Gregersen and P. Krogstrup

Niels Bohr Institute, Rockefeller Komplekset, Juliane Maries Vej 30

Submitted December 2007

ABSTRACT

Aims. To determine the properties, such as the mass and radius, for the two F-star components of the double-lined eclipsing binary star system BK-pegasus. The spectroscopic observations comes from the NOT-telescope on La Palma in August 2007, and the results are compared to predictions from theoretical stellar evolutionary models.
Methods. A series of 13 spectra are observed, were only 11 of them are used in the analysis because of bad quality on two of them. These spectra are separated in time, so we have data points for a complete orbital phase of the system. Cross correlating the spectra to theoretical templates, we determine the radial velocities and from these we determine the parameters of the orbit, which then by some analyses gives us the properties of the components.
Results. We find that the masses and radius of the primary and secondary component of BK Pegasus are respectively: $M_{pri} = 1.4143 \pm 0.0077 M_{\odot}$, $R_{pri} = 1.9901 \pm 0.0068 R_{\odot}$ and $M_{sec} = 1.2691 \pm 0.0094 M_{\odot}$, $R_{sec} = 1.4771 \pm 0.0131 R_{\odot}$.
Conclusions. We compare our results and results from photometrical observations with predictions from recent theoretical evolutionary models. Via isocrone diagrams we see that the results match these evolutionary tracks for ages about 3 Gyr, with an metalicity abundance of approximately $[Fe/H] = 0$.

Key words. spectroscopic binary – stellar models

1. Introduction

Over 7000 eclipsing binaries have been discovered in the local Group over the last decade, and the study of these systems have contributed to a thorough understanding on several aspects of stellar models. One important aspect is the determination of the mass of distant stars. The gravitational pull between them causes them to orbit around their common centre of mass, and from the time variation of the spectrum of these binaries, the masses of its stars can be determined. Now the relation between a star's appearance and its mass can be predicted, which also allows for the determination of the mass of non-binaries. Because a large proportion of stars exist in binary systems, binaries are also particularly important to our understanding of the processes by which stars form. In particular, the period and masses of the binary tell us about the amount of angular momentum in the system. Because this is a conserved quantity, binaries give us important clues about the conditions under which the stars were formed. Furthermore they can be used as primary distance indicators. We want to contribute to this important research by making spectroscopic observations of the double-lined eclipsing binary system called BK Pegasus. From our own spectroscopic observations we determine the masses of its two components. We will furthermore use the results to compare with the predictions of theoretical stellar evolutionary models.

2. Basic theory

This section will focus on the derivation of the fundamental parameters of the binary. The masses of the two components, m_1 and m_2, can be used to test theoretical stellar evolutionary models and hence these will be the main parameters to be determined. To obtain these quantities we will need the *radial* velocities, or the line-of-sight velocities, of each component. In praxis these are determined from the observed spectre by means of cross-correlation. This subject will be discussed in the next section. In the present section we will derive expressions for the radial velocity and the minimum masses of the components.

The figure below shows the relative orbit of a binary located in three dimensions and defined by the angles Ω, i and ω. The mass m_1 is located at the origin, O, of the coordinate system. The mass m_2 is located at the point P_2 at time t in the relative orbit. The observers line of sight is along Oz, from below the figure. The orbit of m_2 is clockwise when seen from

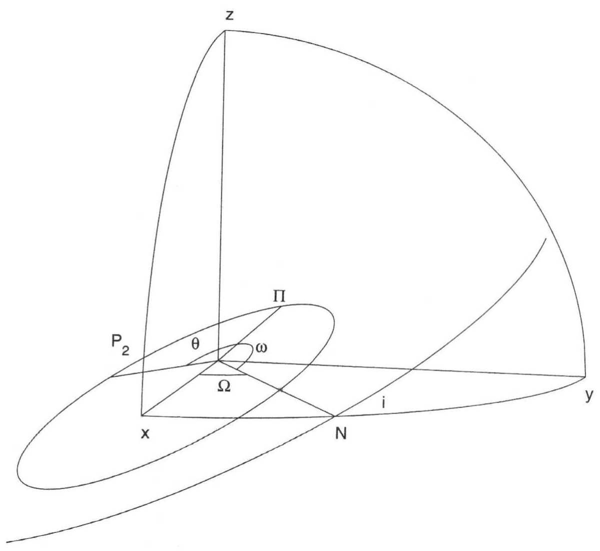

Fig. 1. The relative orbit of a binary in three dimensions. The star with a mass m_1 is situated at the origin while the star of mass m_2 is at the point P_2. The observer views the binary from *below* the figure along the line Oz perpendicular to the tangent plane of the sky denoted by xNy.

the observer. The point Π is called the *periastron* - the exact opposite point on the ellipse is called the *apastron* (this point is not marked in the figure). The angle θ (denoted by $P_2O\Pi$) is related to a quantity called the *phase* of the binary system. The connection will be explained below. Naturally, θ depends on time since the position of the point P_2 does.

The plane defined by xNy is called the tangent sky and is perpendicular to the observer's line of sight. The arc through N is the projection of the orbit onto the celestial sphere. Hence the angle i denotes the *inclination* of the orbit to the tangent sky - when $i = 90°$ the observer will be in the plane of the orbit and will see the two components of the system cover each other at two points in the orbit (though not necessarily at periastron and apastron).

The angle ω is called the longitude of periastron and defines the orientation of the orbit within its own plane. It should be noted that the value of ω determines where on the ellipse the radial velocities (seen from the observer) will be zero. Only if $\omega = 90°$ (or 270°) this will occur at periastron and apastron, and hence in general for an arbitrary ω this will not be the case. The value of ω gives the connection between *phase* of the binary and the angle θ. The phase is defined to be zero when the radial velocities are zero, which means that only for $\omega = 90°$ or 270° will θ =phase. For an arbitrary ω, however, phase = θ + const.

From the figure we see that the polar coordinates of P_2 are $(r, \theta + \omega)$. These can be resolved into two components in the orbital plane: $r\cos(\theta + \omega)$ along the line ON, and $r\sin(\theta + \omega)$ at a right angle to the line ON. When the second component is projected into the line of sight Oz, the following is obtained

$$z = r\sin(\theta + \omega)\sin i \quad (1)$$

By definition the radial velocity is given by $V_{\rm rad} = \frac{\rm d}{\rm dt}z = \dot{z}$ and hence

$$V_{\rm rad} = \sin i[\sin(\theta + \omega)\dot{r} + r\cos(\theta + \omega)\dot{\theta}] \quad (2)$$

The appropriate expressions for $\dot{r}$ and $\dot{\theta}$ are found by use of the polar equation for an ellipse $r = a(1 - e^2)/(1 + e\cos\theta)$ which gives $\dot{r} = e\sin\theta r\dot{\theta}/(1 + e\cos\theta)$, where a is the semi major axis and e is the eccentricity of the ellipse, and Kepler's second law $r^2\dot{\theta} = 2\pi a^2(1 - e^2)^{1/2}/P$, where P is the period. Using these relations we obtain the expression for the radial velocity:

$$V_{\rm rad} = \frac{2\pi a\sin i}{P(1 - e^2)^{1/2}}[\cos(\theta + \omega) + e\cos\omega] \quad (3)$$

This is usually written in the more compact form

$$V_{\rm rad} = K[\cos(\theta + \omega) + e\cos\omega] + \gamma \quad (4)$$

where $K = \frac{2\pi a\sin i}{P(1-e^2)^{1/2}}$ is called the semiamplitude of the velocity curve, and γ is the systemic velocity, or the radial velocity of the centre of mass of the binary system.

It is now a simple matter to find the semi major axes. These are simply found by rearranging the expression for K:

$$a_{1,2}\sin i = \frac{(1 - e^2)^{1/2}}{2\pi}K_{1,2}P \quad (5)$$

To derive the expressions for the minimum masses, $m_{1,2}\sin i$, we use the relation $m_1a_1 = m_2a_2$, and that $G(m_1 + m_2) = 4\pi^2a^3/P^2$. Then, when substituting for m_2, we get

$$(m_1 + m_1a_1/a_2) = 4\pi^2a^3/GP^2 \quad (6)$$

But $a\sin i = a_1\sin i + a_2\sin i$, which are both determinate from before, and hence

$$m_1\sin^3 i = \frac{4\pi^2}{GP^2}\frac{a^3\sin^3 i}{1 + (a_1\sin i)/(a_2\sin i)} \quad (7)$$

We then use the foregoing expressions to obtain

$$m_{1,2}\sin^3 i = \frac{1}{2\pi G}(1 - e^2)^{3/2}(K_1 + K_2)^2K_{2,1}P \quad (8)$$

Note the order of the subscript for the masses and the semi amplitudes.

3. Observations and data reduction

The detached eclipsing binary BKPeg consists of two rather identical F-type stars, R.A (2000) 23 47 08.47 and Decl (2000) +26 33 59.9. The system has been studied before (Popper & Dumont 1977, Popper et al 1981, Popper 1983, Popper & Etzel 1995) with both photometrical and spectroscopic analysis available.

Our data consists of 13 high resolution spectra which were obtained using the cross-dispersed high-resolution echelle spectrograph FIES on the 2.5 m Nordic Optical Telescope (NOT) on La Palma, between the 20th and 24th of august 2007. The entire spectral range 370-740 nm is covered without interruption in a single, fixed setting. The FIES spectrograph is mounted in a building separate from and adjacent to the NOT

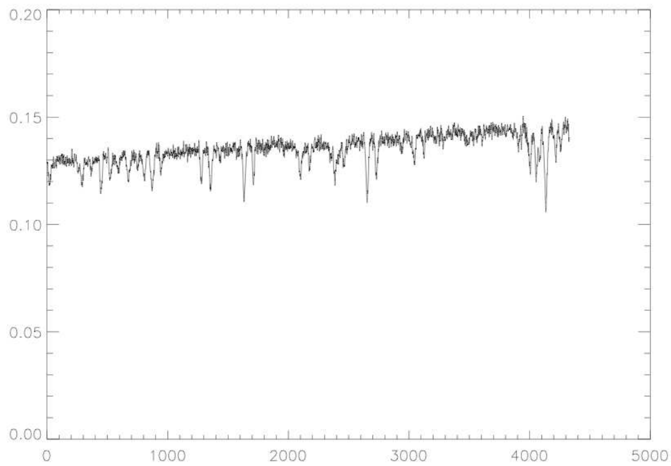

Fig. 2. This is an example of a normal spectrum. The intensity lies in the range 0.1-0.2 and the mean of the values follows a straight line.

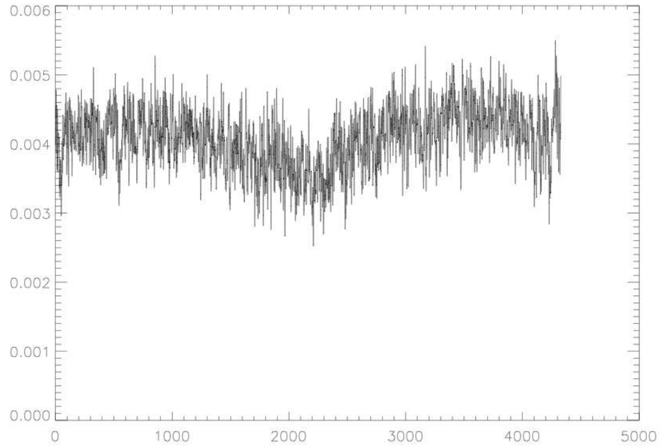

Fig. 3. Abnormal spectrum. The intensity is very low (less than 0.005) and the mean of the values looks like a periodic function.

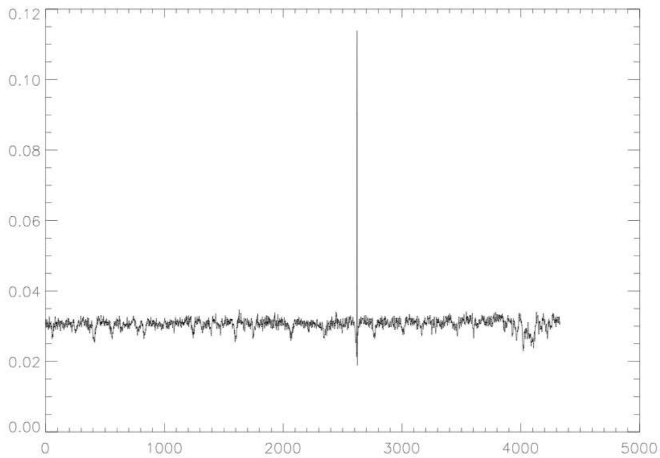

Fig. 4. Abnormal spectrum. The intensity is low (less than 0.04) with a very large reading in the middle.

dome, which is done to avoid sources of thermal and mechanical instability. We used fibre 3 (R=45000) for all exposures. The exposure time was close to 10 minutes for each observation (giving a S/N of approximately 100) with ThAr exposures immediately after each observation for wavelength calibration. A logbook for the spectroscopic observations is shown in table 1, where HJD is the Heliocentric Julian Day and t_{exp} the exposure time.

The FIEStool package, prepared by Soren Fransen, was used for the basic data reduction[1]. Since the ThAr spectrum used for the reduction was a standard spectrum we had to cross correlate it with the ThAr spectra we had made after each observation. For this we used the program cc-thar.pro. The program loads the standard ThAr spectrum and the ThAr spectrum obtained after the observation and then uses the program ccf.pro to cross correlate. Since different ThAr spectra was used for the reduction of the data collected on the Ph.d-nights, the program had to be modified to contain different standard spectra. The corresponding corrections in velocity were saved in a file for later use.

Comparing the images of the 13 spectra it is clear that 2 of them stand out (figure 2, 3 and 4).

They have an obviously different shape (especially the first) and both of them have a very low intensity compared to the other spectra. We don't know the reason for this discrepancy, but it is certain that it is an observation error and has nothing to do with the behaviour of the star system. In the following data analysis we will therefore only deal with the 11 normal spectra.

3.1. Cross correlation

Cross correlation is a procedure that determines the shift in wavelength due to the radial velocities of the components relative to the overall motion of the centre of mass (i.e a Doppler shift).

If we can identify the absorption lines in the observed spectrum we can compare it with similar lines in a spectrum of a similar star which is not Doppler shifted or where the shift is known (this spectrum is called a template). Then, when placing the two spectra on top of each other so the wavelength axes coincide, the distance, $\Delta\lambda$, between a line in the observed spectrum and the corresponding line in the template will be a measure of the Doppler shift, i.e. the radial velocity. The cross correlation formula is given by

$$c(z) = \int_{-\infty}^{\infty} f^*(x)g(x+z)dx \tag{9}$$

where f and g are complex functions of the reel variable x. For each value of the independent variable z the cross correlation function, $c(z)$, returns a value that tells us how well f and g coincide—the larger the value the better a match. In our case these functions will correspond to the observed spectrum and the template and hence z in equation (9) will be a measure of the Doppler shift.

As we see from equation (9) the special feature with cross correlation is that we not only compare *one* line in the ob-

[1] See the NOT website for details:
http://www.not.iac.es/instruments/fies/fiestool/FIEStool.html

Table 1. Observation log

HJD minus 2400000	Orbital phase	t_{exp} / sec
54333.45954	0.66595	300
54333.46395	0.66675	300
54334.42028	0.84095	300
54335.52798	0.04272	300
54335.53871	0.04468	600
54335.62202	0.05985	600
54335.72880	0.07930	600
54336.41868	0.20497	600
54336.43502	0.20794	600
54336.52436	0.22422	600
54336.74165	0.26380	600
54337.44771	0.39241	600
54337.58944	0.41822	600

served spectrum with the corresponding line in the template—an entire order of the observed spectrum consisting of multiple lines is compared to the template. However, to do this we need first to rebin the wavelengths to logarithmic scale. The reason is that $\Delta\lambda$ depends on the particular wavelength of the absorption line we consider—hence the shift, $\Delta\lambda$, becomes larger when λ increases and it will be impossible to make the all the lines of the considered spectrum coincide with the corresponding lines in the template at the same time. However, if we rebin to log scale so that $\Delta \ln\lambda = \ln\lambda_{\rm obs} - \ln\lambda_{\rm em}$ and use that the standard relativistic Doppler shift is given by $(\lambda_{\rm obs} - \lambda_{\rm em})/\lambda_{\rm em} = [(1+\frac{v}{c})/(1-\frac{v}{c})]^{1/2} - 1 \approx v/c$ for $v \ll c$, i.e. $\lambda_{\rm obs} \approx (v/c + 1)\lambda_{\rm em}$, this dependence disappears:

$$\Delta \ln\lambda \;\approx\; \ln\left(\frac{v}{c} + 1\right) \tag{10}$$

where v is simply the relative velocity between the star emitting the photons and the observer.

Before doing the cross correlation we also need to transform the wavelengths to the barycentric frame. This was done with the program prep-merge.pro. The program reads information about the observation date from the header and then uses this to determine the Julian date and the heliocentric Julian date at mid-exposure. With this information it uses the program bary.pro to calculated the barycentric velocity correction. This new information is added to the header.

The spectra are then cross correlated (one dimensional) with synthetic templates generated from the data obtained by Popper 1983. This is done with the program cc-bkpeg.pro. First of all the program loads the velocity corrections due to the difference in the ThAr spectra and the list of observed spectra with new header information. Since we had several synthetic templates it then asks which one to use. Before the cross correlation itself can take place the spectra needs to be normalised[2]. This is done in the program by fitting a polynomial to each spectrum and then dividing the spectrum with the polynomial. The program then does the cross correlation using the program ccf.pro and saves the output in an SBOP formatted file.

[2] Less light is reflected towards the ends of each order on the CCD when using an Echelle spectrograph. This results in the continuum to be non-linear in wavelengths. Though we analyse the *merged* spectra and hence in principle are past this problem we wish to have the opportunity to fine tune the normalisation.

Since the spectral lines of stars are often more or less broadened due to rotation we also apply a rotational broadening of the template spectrum with the possibility of choosing $V_{rot}\sin i$ among the values 0, 6, 12, 15 and 18 km/h. In general we get the best fit for with $V_{rot}\sin i = 12$km/h for one component and $V_{rot}\sin i = 18$km/h for the other.

For old binary systems the rotational velocity of the two components will synchronise—if the stars are of equal size this velocity will be the same. We have a strong indication that the system is old since the orbits are circular (Popper et al 1981). Hence the fact that the rotational velocities are not the same suggests that the stars are not equal in size.

3.2. Masses from radial velocities

The radial velocities for each component can now be plotted against phase. Theory tells us that the two sets of data points should be fitted with

$$V_{\rm rad} = K[\cos(\theta+\omega) + e\cos\omega] + \gamma \tag{11}$$

where θ equals the phase up to a constant. We remind ourselves that the rest are constant parameters to be determined: $K = 2\pi a \sin i/(P(1-e^2)^{1/2})$ is the semi-amplitude, a the semi-major axis, e the eccentricity of the relative orbit, P the period of the orbit. γ is systemic velocity i.e. the centre of mass velocity, while ω is a quantity which defines the orbits orientation within its own plane. In principle it is now possible two use any fitting programme to fit the data points and get the values of these parameters. These values along with the knowledge of the period, P, then allow us to calculate the *projected* semimajor axes through

$$a_{1,2}\sin i = \frac{(1-e^2)^{1/2}}{2\pi} K_{1,2} P \tag{12}$$

where the indices (1,2) refers to the components.

We get the masses from

$$m_{1,2}\sin i = \frac{1}{2\pi G}(1-e^2)^{3/2}(K_1+K_2)^2 K_{2,1} P \tag{13}$$

And since BKPeg is an eclipsing binary, i.e. the orbits are in a plane containing the line of sight, $i \simeq 90^\circ$ and hence $\sin i \simeq 1$. The exact angle, i, is found from photometrical observations of the system. Popper et al 1981 have results from such observations, which means we are able to calculate the absolute masses and semimajor axes.

First, however, we need to determine the parameters mentioned above. For this purpose we use the programme asbop.pro which (from an IDL-prompt) runs the fortran programme SBOP (Spectroscopic Binary Orbit Programme). SBOP uses a solution method known as *diffential corrections* which is based on a first-order Taylor series expansion about a preliminary solution given by equation (11). If we identify the radial velocity as a function of the parameters K, e, ω, T, P [3]

[3] T and P enter via the true anomaly θ

and γ, i.e. $V_{\rm rad} = F(K, e, \omega, T, P, \gamma)$, then to first order

$$\Delta V_{\rm rad} = \sum_i \frac{\partial F}{\partial x_i} \Delta x_i \tag{14}$$

where the x_i's are K, e, ω, T, P and γ. Since F is analytic the partial derivatives can be derived to give an explicit expression for $\Delta V_{\rm rad}$—this result was first shown by Lehmann-Filhés in 1908.

Now, if we have preliminary estimates of K, e, ω, T, P and γ we can calculate $V_{\rm rad}$ at some time t through equation (11).[4] We then identify $\Delta V_{\rm rad}$ with the difference between the observed velocity and the calculated velocity. This leaves us with an equation (equation (14)) consisting of 6 unknowns, namely ΔK, Δe,... i.e. the differential correction terms. However, since this equation can be written for each of the n observations we have n equation with 6 unknowns. Thus we need to have at least 6 observations (and in praxis substantially more to over-determinate the set of equations) to calculate the differential correction terms.

The procedure is then quite simple: Solving the n equations yields a first estimate of the 6 differential correction terms. We then put $K_1 = K_0 + \Delta K$ and so forth. These new values are then used for calculating a new $V_{\rm rad}$ and the foregoing procedure is repeated giving new estimates of the differential correction terms. The iteration is stopped when the least-squares solution is reached yielding the best values for the parameters K, e, ω, T, P and γ.

We choose to fix the the period, P, since this is known from photometrical observations (Popper et al 1981). Photometrical observations give the period with higher accuracy than can be determined from spectroscopic observations because photometrical variations are more sharply defined than the velocity variations (Hilditch 2001). We also fix the systemic velocity, γ, the semi-amplitudes K_1 and K_2 and the time of periastron passage, T. The rest of the parameters, i.e.e and ω are set to be free parameters to be altered by SBOP until the best fit is achieved.

3.3. Systematic errors in radial velocities

Systematic errors can occur of a variety of reasons. The most important, and potentially most difficult, involves blending of the spectral lines. The way two particular lines blend depends on their amplitudes, how much they are broadened and on the spectral resolution employed. Particularly, if the spectral resolution is not high enough to separate the lines from the two components the position of the lines will be *dragged* from their true velocities towards the systemic velocity of the binary (Hilditch 2001). The result will be that the determinations of the semi amplitudes will be smaller than their true values, and hence, through equation (13), the masses will be seriously under-estimated. Fortunately, it seems that resolution used is sufficient to separate the lines. However, blending of the spectral lines *do* occur to some extent and we wish to eliminate the errors this may introduce.

To correct for systematic errors we construct *one* new template containing the spectra of *both* stars. This is done using the program mkeb.pro. The program combines the information about the two stars given by SBOP with two synthetic templates to create a synthetic spectrum that looks like the star system. Since the two stars have different rotation velocities two different templates are used: one with rotational velocity 12km/s and one with rotational velocity 18km/s. This synthetic spectrum will depend on the phase and hence we need to make a synthetic spectrum for each observation. We then cross correlate our new synthetic spectra with the original template (the one with rotational velocity 15km/s). For this we used a modified version of the program cc-bkpeg.pro, since we no longer need a normalisation of ThAr correction of the spectra. If there are no systematic errors this correlation will give the same radial velocity as previously found. However, this will probably not be the case having the reason for the systematic errors in mind. We now run SBOP with every option fixed such that the programme only fits a velocity curve through the data points, but doesn't calculate the masses, semi-major axes etc. The final output file then contains the velocities based on our observed spectra and the velocities based on the synthetic spectra. The difference of the two is due to systematic error.

3.4. Final result for the elements of orbit

We now have the final results for the minimum masses, semi amplitudes and semi major axes. They are listed in table 2.

Table 2. Minimum masses, $m \sin i$, semiamplitudes K and semi major axes $a \sin i$ for BKPeg.

	Primary	Secondary
$m \sin i$	$(1.411 \pm 0.006) M_\odot$	$(1.267 \pm 0.006) M_\odot$
K	(79.2705 ± 0.2211)km/s	$88.3326 \pm 0.2211)$km/s
$a \sin i$	$(5.984 \cdot 10^6 \pm 1.6 \cdot 10^4)$km	$(6.668 \cdot 10^6 \pm 1.6 \cdot 10^4)$km

These seem to be in good agreement with the results obtained by Popper 1983, who calculated the masses to be $1.43 M_\odot$ for the primary component and $1.28 M_\odot$ for the secondary component.

Figure 5 shows the velocity plotted against phase along with the calculated curve. The two plots in the bottom of the figure show the deviations from the curve. The points marked by squares represent the abnormal spectra and are given zero weight in the calculations.

The velocity of the system was found to be -7.0013 km/s (± 0.1178) and the ratio between the masses to be M(2)/M(1) = 0.897 (± 0.003).

[4] It should be mentioned that we need to solve Kepler's equation for the *eccentric anomaly* E, $E - e \sin E = 2\pi(t - T)/P$, and then calculate the true anomaly θ through the relation $\tan(\theta/2) = [(1 + e)/(1 - e)]^{(1/2)} \tan(E/2)$. This enables us to calculate $V_{\rm rad}$

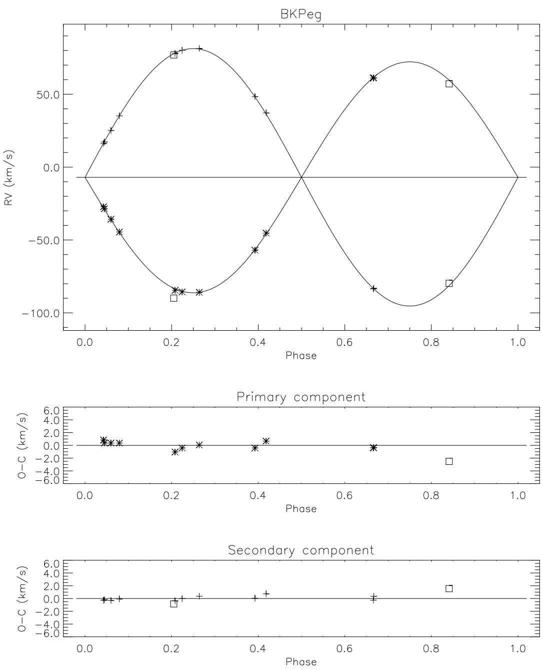

Fig. 5. Plot of velocity vs. phase. The white squares represent the two abnormal spectra which has been given zero weight in the calculations.

3.5. How to improve the results

There are a couple of obvious choices we can make to improve our results. They concern the way we calculate the radial velocities. We have used one-dimensional cross correlation, but there are two other, and possibly better, ways to calculate the radial velocities. The first is a natural extension of the one-dimensional cross correlation, namely the two-dimensional cross correlation (Zucker & Mazeh 1994).

The procedure of two-dimensional cross correlation includes *two* templates simultaneously (one for each of the components) with a given flux-density ratio equal to the that between the two stars in the observed binary. If this ratio is not known it can be set as a parameters to be determined in the correlation.[5]

Two dimensional cross correlation is probably the most used method for calculation radial velocities of spectroscopic binaries. Zucker & Mazeh 1994 have demonstrated that this method is extremely effective in determining the radial velocities of both stars when the velocity separation is quite small. According to Zucker and Mazeh both one- and two-dimensional cross correlation give the same result at separations above 70-80 km/s, but the two-dimensional technique can continue down to separations of 20 km/s and less. As we have seen, the semiamplitudes of the components of BKPeg are $\sim$ 80km/s and $\sim$ 90km/s for the primary and secondary component, respectively, so it seems the one-dimensional cross correlation should be efficient most of the time.

[5] The mathematical details on the subject of two-dimensional cross correlation can be found in Hilditch 2001.

Another method of calculating the radial velocities is called *spectral disentangling* (Simon & Sturm 1994). This method uses the techniques of cross correlation and *tomography* simultaneously.

To do tomography we need to find a *mean* spectrum of each component. A set of spectra will normally comprise about $m \sim 20 - 30$ observations. If the individual radial velocities have been determined by cross correlation it is possible to Doppler-shift all spectra to the rest frame and separate the spectra of the two components from the combined spectrum. The resulting two spectra are then the *mean* spectra of the components and should be of high quality, because they would be additions of all m spectra in the sample. The S/N ratio should be of the order $m^{1/2}$ greater than with a single observation.

Now, we could think of the procedure of finding the radial velocities as a more iterative one, with preliminary templates adopted from consideration of the spectral types of the stars in the binary, together with test runs to find the best separated peaks in the cross correlation function, which would lead to a first set of radial velocities for each component. These velocities could then be used in a tomography code to extract the best mean spectrum for each star. These mean spectra would serve as the templates in the next iteration through the original set of spectra to achieve more accurate radial velocities and more refined mean spectra via tomographic separation.

Southworth & Clausen 2007 shows that the method of spectral disentangling constrained by the results of Gaussian fitting gives the most reliable radial velocities of the star system DW Carinae in the open cluster Collinder 228.

For the technique of spectral disentangling to be really successful, the spectra need to have a high S/N and be well distributed around the orbital cycle. The last requirement is often the hardest to achieve since it relies on how much observation-time the observer is granted. In our case the coverage of the phase is probably a bit low and we only have 11 useable observations. Hence it seems we have little to gain by using this method.

4. Stellar evolutionary models

In the present section we will show how the results obtained in the previous section can be used to test theoretical stellar evolutionary models.

All stars formed from the same interstellar gas can be assumed to have the same age and chemical composition. This information can be used to test stellar evolutionary models. However, first we need to know the effective temperatures, the luminosities of the stars and the inclination of the orbit (in order to calculate the absolute masses and semimajor axes).

This is achieved from solutions of light curves. However, the analytical theory used to determine the radii and the inclination requires a detailed description which seems to be a bit misplaced at this point (also because we have not performed the photometrical observations ourselves but only use the results from it)[6]. Instead, we now present a way to estimate the radius of the stars which hopefully leaves the impression that light curves can in fact be used to determine these parameters.

A photometrical light curve for an eclipsing binary will show minima in the total flux from the system when the two components cover each other—these are called the *annual transit* (when the smaller component, the secondary, passes in front of the larger, the primary) and the *total occultation* (when the larger passes in front of the smaller). We now consider the case where the orbit is circular. We label the phase where the secondary component exactly begins to pass in front of the primary component, Φ_1; the phase where the secondary component exactly is within the disc of the primary component, Φ_2; the phase where the secondary component exactly begins to exit the disc of the primary component, Φ_3; and the phase where the secondary component exactly has left the disc of the primary component, Φ_4. We then see that

$$\frac{\Phi_2 - \Phi_1}{\text{total orbital phase}} = \frac{\Phi_4 - \Phi_3}{\text{total orbital phase}} = \frac{2R_2}{2\pi a} \tag{15}$$

where R_2 is the radius of the secondary component and a is the separation of the stars (a is related to the barycentric semi major axes through $a = a_1 + a_2$). Now, the "total orbital phase" is equal to 1 by definition (i.e. the primary eclipse occur at phase=0 or 1 while the secondary eclipse occur at phase=0.5). Hence we get

$$\Phi_2 - \Phi_1 = \Phi_4 - \Phi_3 = \frac{2R_2}{2\pi a} \tag{16}$$

Similarly, we get the radius of the primary component by considering the phase differences, $\Phi_3 - \Phi_1$ and $\Phi_4 - \Phi_2$. We get

$$\Phi_3 - \Phi_1 = \Phi_4 - \Phi_2 = \frac{2R_1}{2\pi a} \tag{17}$$

Since the phases are measured directly and a is known from the spectroscopic observations the only unknowns in the equations above are $R_{1,2}$. Thus, the equations allow us to calculate the absolute radii of the components.

However, these are just estimates (remember, that we assumed the orbit to be circular). When performing the full analysis as described by Hilditch 2001 we obtain $(i, r_{1,2}, T^{\text{eff}}_{1,2})$. The absolute radii or dimensions of the star are then given by $R_{1,2} = ar_{1,2}$. One of the temperatures, usually T_1, will have been adopted from other data, e.g. such as *de-reddened colour indices* at a total eclipse or using several values of de-reddened colour indices at different orbital phases. The other temperature, T_2, may have been determined from the light curve solutions directly. The two temperatures can also be determined from other techniques, but we shall not mention this subject further here.

To establish the total luminosity of each star we use the relation $L_{1,2} = 4\pi R^2_{1,2}\sigma T^4_{1,2}$, where the radius is taken to be the so-called volume radius, i.e. the radius of a sphere with the same volume as the (commonly) non-spherical star (Hilditch 2001).

[6] A thorough derivation of the orbital inclination and the radii of the two stars is given in Hilditch 2001 on pp. 226-236

4.1. Isochrones

We can now plot the two components of the binary in a HR-diagram with, for example, $\log L_s$ and $\log T_{\text{eff}}$ on the axes. In a HR-diagram a star can be described by a point P in the ($\log T_{\text{eff}}$, $\log L_s$) plane. P can be regarded as a function of the mass and the age of the star. An *evolutionary track* is a curve in the plane consisting of a collection of points corresponding to a particular mass but different ages, whereas an *isochrone* is a collection of points corresponding to the same age but different mass (Christensen-Dalsgaard 2006).

Since the two components of the binary are assumed to have the same age (and chemical composition) they should lie on the same isochrone.

We could also choose to show the isochrones in a mass-luminosity plot or in a temperature-radius plot (which we will do below).

4.2. Comparison with evolutionary model

We are now ready test a model. We have used a model by Demarque et al 2004. This model includes an improved core overshoot treatment and covers the case of no convection in the core to a fully developed convective core overshoot. Hence this model is an improvement of previous models and has been shown to give good results (Demarque et al 2004)[7].

Figure 6 shows a mass-luminosity plot with theoretically calculated isochrones along with two points with errorbars representing the components of BKPeg. The solid lines are isochrones for 0.5, 1, 2, 3 and 4 Gy. The metallicity $[Fe/H]$ gives the logarithm of the ratio of the star's iron abundance compared to that of the Sun, i.e.

$$[Fe/H] = log_{10}\left(\frac{N_{Fe}}{N_H}\right)_{star} - log_{10}\left(\frac{N_{Fe}}{N_H}\right)_{Sun} \tag{18}$$

where N is the number of atoms per unit volume. In this equation "iron" actually cover every other element than hydrogen and helium.

Thus, $[Fe/H] = 0$ means that the iron abundance is the same as for the Sun. From figure 6 we see that the two components are close to the 3 Gy isochrone and hence predicts this age for the binary.

Figure 7 shows the same plot as figure 6 but for $[Fe/H] = -0.09$, i.e. for a slightly lower metal abundance than for the Sun[8]. In this plot the two components of BKPeg lie close to the 2 Gy isochrone.

To find out which of the two metal abundances that gives the best match we now plot mass against radius of the stars. Figures 8 and 9 show the plots for $[Fe/H] = 0$ and $[Fe/H] =$

[7] For further information about this model we refer to the paper by Demarque et al 2004 which is listed in the reference list. We also note that the plots in this section were made by Jens Viggo Clausen, Niels Bohr Institute, University of Copenhagen. I.e. we provided him with the results obtained from SBOP in the previous section, then he used these results along with the photometrical results from Popper & Etzel 1995 to calculate the isochrones using the the theoretical stellar evolutionary model by Demarque et al 2004.

[8] To be exact: $10^{-0.09} \simeq 0.8$ times the iron abundance in the Sun.

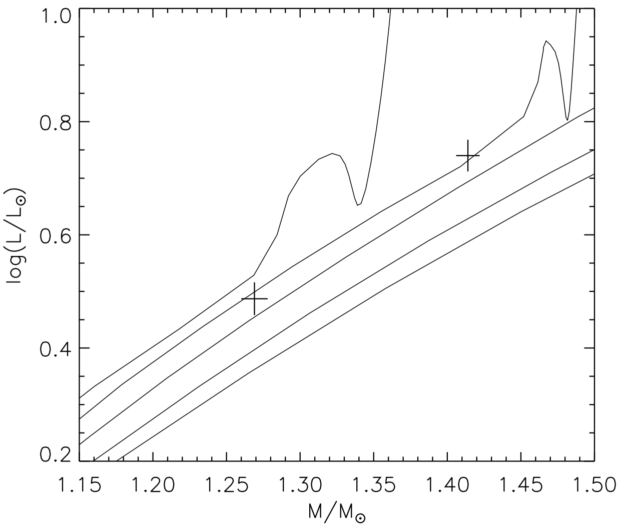

Fig. 6. Mass-Luminosity plot. Metallicity [Fe/H]=0. The solid lines are isochrones with ages 0.5, 1, 2, 3 and 4 Gy.

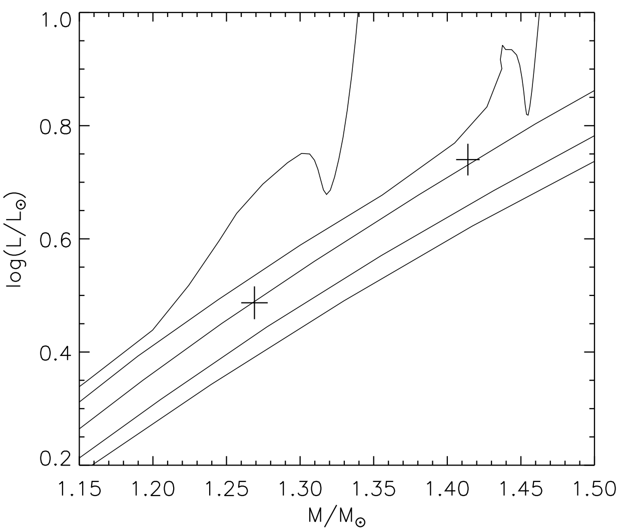

Fig. 7. Mass-Luminosity plot. Metallicity [Fe/H]=-0.09. The solid lines are isochrones with ages 0.5, 1, 2, 3 and 4 Gy.

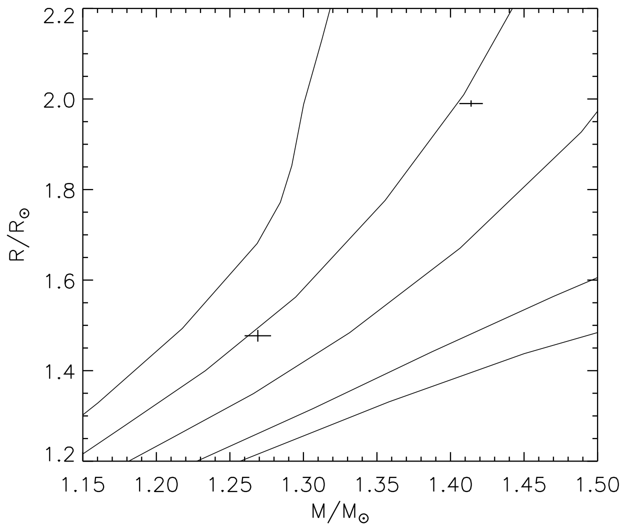

Fig. 8. Mass-radius plot. Metallicity [Fe/H]=0. The solid lines are isochrones with ages 0.5, 1, 2, 3 and 4 Gy.

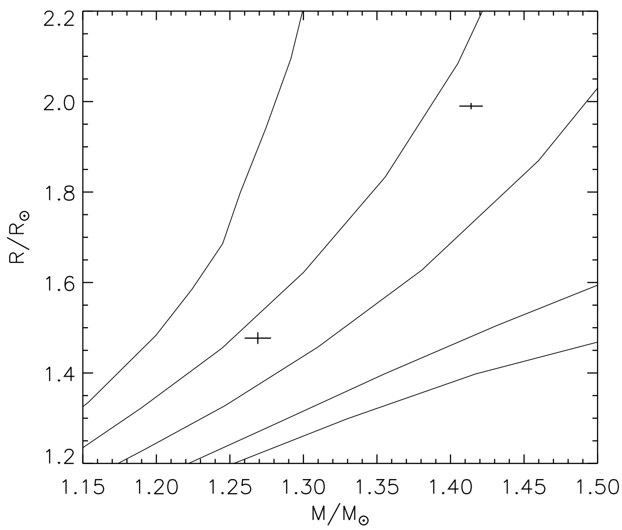

Fig. 9. Mass-radius plot. Metallicity [Fe/H]=-0.09. The solid lines are isochrones with ages 0.5, 1, 2, 3 and 4 Gy.

−0.09, respectively . We see that the first plot, with [Fe/H]=0, predicts an age of 3 Gy—the same prediction as found in the mass-luminosity plot for this metallicity. The second plot, [Fe/H]=-0.09, predicts an age that is closer to 3 Gy than to 2 Gy contrary to the corresponding mass-luminosity plot (figure 7). Thus, so far it seems the model used fits our stars if they are approximately 3 Gy and have [Fe/H]=0.

Finally, we also show two diagrams, one for [Fe/H]=0 and the other for [Fe/H]=-0.09, with $T_{\rm eff}$ plotted against radius. These plots are a bit more complicated to interpret since we now need to show lines for both stars of the same mass and stars of the same age (isochrones). In the plots we have once again showed the isochrones (now as dashed lines) for 0.5, 1, 2 ,3 and 4 Gy. Also we show lines (solid lines) of constant mass fixed to be the ones calculated for each of the components of BKPeg. The two points with error-bars represent the components of BKPeg. Thus, we want these points to lie on the intersection between one of the isochrones and the corresponding mass lines—i.e. the two points should be on the same isochrone, but also on their corresponding mass line. (Fortunately,) the tendency from the first four plots continue. It seems the figure 11 shows a better agreement between the points and the intersections than figure 10. Thus, we conclude that the model by Demarque et al 2004 agrees with our data if the components of BKPeg have approximately the same metallicity as the Sun and ages of about 3 Gy.

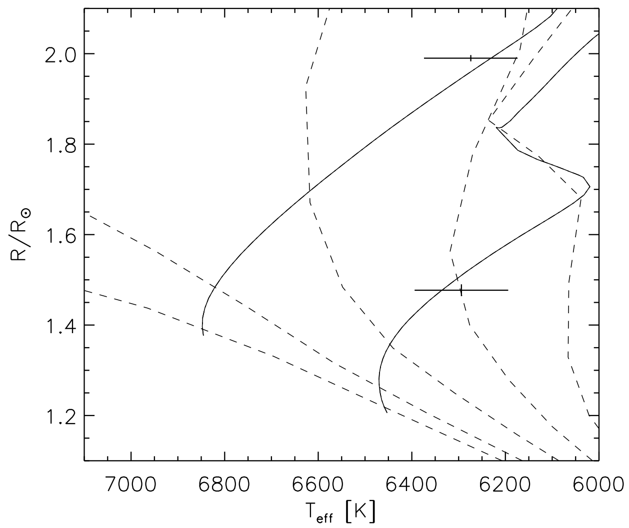

Fig. 10. Temperature-radius plot. Metallicity [Fe/H]=0. The dashed lines are isochrones with ages 0.5, 1, 2, 3 and 4 Gy. The solid lines are constant mass lines fixed to be the calculated masses of the components of BKPeg

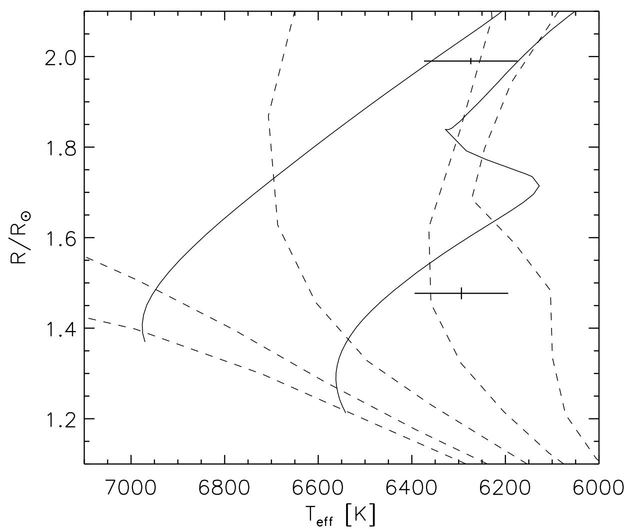

Fig. 11. Mass-radius plot. Metallicity [Fe/H]=-0.09. The dashed lines are isochrones with ages 0.5, 1, 2, 3 and 4 Gy. The solid lines are constant mass lines fixed to be the calculated masses of the components of BKPeg

5. Conclusions

The general technique of finding radial velocities for double-lined eclipsing binary star systems presented in this paper have been used by many astronomers, and has given us a good insight into the porperties of stars. Of course there are still many unanswered questions and uncertainties in the work when the results are compaired with theory, but as more advanced and precise techniques in the data analysis are been developed it will give more precise answers, and it will hopefully improve our understanding and theory.

Table 3. Minimum masses, $m \sin i$

Primary	Secondary
$(1.411 \pm 0.006)M_\odot$	$(1.267 \pm 0.006)M_\odot$

The result for the minimum masses are listed in table 3. These seem to be in good agreement with the results optained by Popper 1983. We have also used our results together with the results from photometric observations (Popper et al 1981) to test a stellar evolutionary model by Demarque et al 2004 and find that the model fits our data for a metallicity $[Fe/H] \simeq 0$ and stellar ages of $\sim$3 Gy.

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